

III. "On the Number of Figures in the Period of the Reciprocal of every Prime Number below 20,000." By WILLIAM SHANKS. Communicated by the Rev. GEORGE SALMON. Received December 2, 1873*.

The following Table, in reality the joint production of the Rev. George Salmon, F.R.S., and myself, was commenced, and indeed nearly completed, before either calculator was distinctly aware that Burckhardt, Jacobi, or Desmarest had written or published any thing on the same subject. This fact is perhaps to be regretted; but it has led to the independent recalculation, by two different methods, both of Burckhardt's (Jacobi's Table is professedly a reprint of Burckhardt's) and of Desmarest's Table, and has resulted in the detection of several errors, which have, as far as I know, never before been pointed out. These errors, in the first place regarded as discrepancies, have been carefully examined; in fact every case has been reworked by me, with the view of either proving or disproving the accuracy of such numbers as differ from those in our Table. The result is, that such discrepancies are found to be errors both in Burckhardt and Desmarest. The two lists of errors are given below.

I now proceed to give the theorems used, and some account of the means employed by me in forming the Table.

Let P be any prime number, except 2 and 5. Then, from Fermat's theorem, we have $\frac{10^{P-1}}{P} \equiv 1$; or, adopting the usual notation, $10^{P-1} \equiv 1$.

Again, since the number of figures in the period of the reciprocal of all primes is not $P-1$ (or, in other words, since 10 is not a primitive root of all primes),

Let $10^{\frac{P-1}{n}} \equiv 1$, where n is even or odd, not less than 2, and not greater than $\frac{P-1}{2}$. Then we have

(1) The number of figures in the period of the reciprocal of P is either $P-1$ or a submultiple of $P-1$.

(2) Let a and b be integers, and let m be the remainder from $\frac{10^a}{P}$; that is, let $10^a \equiv m$; then $10^{ab} \equiv m^b$.

In practice b is never greater than 2, at least little or no advantage is gained by putting b higher. Also ab need not be greater than $\frac{P-1}{2}$.

Cor. When m is greater than $\frac{P-1}{2}$ we may obviously use $P-m$, or simply $-m$; for $(P-m)^2 = P^2 - 2Pm + m^2 \equiv m^2$, or, because $(-m)^2 = m^2$, b being 2.

(3) Let $10^a \equiv m$, and $10^b \equiv n$; then $10^{a+b} \equiv mn$.

In practice $a+b$ is never greater than $P-1$.

Cor. 1. When m and n are each of them less than P , we may with advantage use $-m$ and $-n$; that is, we may subtract m and n severally from P ; for $(P-m)(P-n) = P^2 - P(m+n) + mn = (-m)(-n)$.

Cor. 2. When m is $> \frac{P-1}{2}$, and n is $< \frac{P-1}{2}$, or *vice versa*, we may use

* The part from 17,000 to 20,000 was received January 8, 1874.

— m and n , or *vice versâ*, obtaining a negative result, which becomes positive by being subtracted from P .

(4) Let $2c$ and $3c$, not greater than $\frac{P-1}{2}$, be submultiples of $P-1$; and let $10^c \equiv \pm S$, and $10^{3c} \equiv S-1$; then $10^{3c} \equiv \mp 1$. This is evident from (2) and (3).

From (1) we have $10^{\frac{P-1}{2}} \equiv \pm 1$, according as the submultiple of $P-1$ is even or odd.

On these theorems and adjuncts my calculations have been based. They enable us to find the remainder either from $\frac{10^{P-1}}{P}$, or from any submultiple, such as $\frac{10^{\frac{P-1}{n}}}{P}$, or from any figure in $\frac{10^{P-1}}{P}$, and, if required, the *figure* itself. Compared with other methods, such for instance as Dr. Salmon's *, mine may seem tedious, requiring as it does much multiplication and division. All I can say is, I did not find it so, though I am free to admit that the calculation of such a Table as ours demands very considerable labour.

It would be foreign to my purpose to enter upon the consideration of primitive roots, or even of prime numbers. If we have found 10 to be a primitive root of a great many prime numbers between 10,000 and 20,000, we have contributed something, as far as I know, quite new. In addition to this we have found the number of figures in the period of each of the other primes between 10,000 and 20,000, and have corrected upwards of 70 errors in Burckhardt's and Desmarest's Tables.

I beg to refer to the works of Euler, Lagrange, Legendre, Gauss; Poincot, Cauchy, and Jacobi (mentioned by Desmarest), and to Desmarest himself, for valuable information touching prime numbers and primitive roots.

I cannot, however, refrain from quoting from Desmarest's 'Théorie des Nombres' the view of Euler as to prime numbers and primitive roots:—"On ne peut saisir entre un nombre premier et les racines primitives qui lui appartiennent, aucune relation d'où l'on puisse déduire *une seule* de ces racines, de sorte que la loi qui règne entre elles paraît aussi profondément cachée que celle qui existe entre les nombres premiers eux-mêmes."

Not discouraged by Euler's remark, Desmarest thus writes:—"Car pourquoi nous serait-il défendu d'ajouter que nous croyons que l'intelligence humaine n'a pas, sur ce point, dit son dernier mot, et que les opérations nombreuses que nous avons dû faire sur les nombres, ne nous ont pas convaincu de l'impossibilité de saisir, sinon l'ensemble, du moins quelques-uns des anneaux de la chaîne mystérieuse qui unit les racines primitives aux nombres premiers."

* *Note by Dr. Salmon.*—The method here referred to is explained, 'Messenger of Mathematics' (1872), p. 49. It is founded on the remark that if we have $10^a \equiv 2^p$, $10^b \equiv 2^q$, we may deduce $10^{aq-bp} \equiv 1$. Thus, let the prime be 251, we can at once write down the equations $10^3 \equiv -2^2$, $2^a \equiv 10^1$, whence immediately $10^{23} \equiv -1$, $10^{50} \equiv 1$. In like manner from the equations $10^a \equiv 2^3p$, $10^b \equiv 2^m3q$, $10^c \equiv 2^n3^r$, we deduce that the number of figures in the period of the reciprocal of the prime is

$$a(nr-nq) + b(np-rl) + c(lq-mp).$$

By the application of these principles I calculated the results obtained in the following Table as far as 18500. For the primes above that number Mr. Shanks is solely responsible; but my experience of his accuracy gives me confidence in his results.

TABLE I. List of Errors in Desmarest's Table.

| Primes. | No. of figures in period of reciprocal. | | No. of figures in period of reciprocal. | Primes. | No. of figures in period of reciprocal. | | No. of figures in period of reciprocal. |
|----------------|---|-----------|--|-------------|---|-----------|--|
| 3 omitted } | omitted | should be | 1 | 5557 | 1389 | should be | 926 |
| 277 | 138 | " | 69 | 5779 | 5778 | " | 2889 |
| 317 | 158 | " | 79 | 5827 | 582 | " | 2913 |
| 397 | 198 | " | 99 | 6101 | 3050 | " | 1220 |
| 449 | 224 | " | 32 | 6277 | 3138 | " | 1569 |
| 787 | 786 | " | 393 | 6287 | 3143 | " | 6286 |
| 1409 | 1408 | " | 32 | 8421 | | | |
| 1657 | 276 | " | 552 | should be } | | | |
| 1733 | 433 | " | 866 | 6421 | | | |
| 1889 | 59 | " | 118 | 6781 | 6780 | " | 1356 |
| | $\frac{P-1}{8} \& \frac{P-1}{9}$ | " | 298 | 6997 | 3498 | " | 1749 |
| 2087 | | " | | 7001 | 3500 | " | 1750 |
| 3253 | 271 | " | 542 | 7127 | 509 | " | 1018 |
| 3373 | 562 | " | 843 | 7481 | 3740 | " | 748 |
| 3413 | 853 | " | 1706 | 7561 | 3780 | " | 1890 |
| 3517 | 1758 | " | 879 | 7717 | 3858 | " | 1929 |
| 3541 | 60 | " | 20 | 7741 | 2580 | " | 860 |
| 3547 | 3546 | " | 1773 | 7841 | 392 | " | 56 |
| 3637 | 3636 | " | 909 | 7853 | | | |
| 3677 | 919 | " | 1838 | omitted } | omitted | " | 3926 |
| 3769 | 942 | " | 1884 | 8011 | 1335 | " | 2670 |
| 3821 | 1910 | " | 3820 | 8087 | 4043 | " | 8086 |
| 3911 | 3910 | " | 1955 | 8093 | 8092 | " | 4046 |
| 4049 | 1012 | " | 2024 | 8101 | 8100 | " | 1620 |
| 4167 | | | | 8219 | 4109 | " | 8218 |
| should be } | | | | 8423 | { 8422 } | " | 8422 |
| 4157 | | | | | { 4211 } | " | |
| 4397 | 157 | " | 314 | 8521 | 355 | " | 710 |
| 4621 | 4620 | " | 924 | 8609 | 538 | " | 1076 |
| 4651 | 2325 | " | 4650 | 8681 | 4340 | " | 868 |
| 5871 | | | | 8893 | 4446 | " | 2223 |
| should be } | | | | 8999 | 8998 | " | 4499 |
| 4871 | | | | 9067 | 9066 | " | 4533 |
| 4943 | 2471 | " | 4942 | 9187 | 9186 | " | 4593 |
| 5081 | 2540 | " | 1270 | 9397 | 4698 | " | 81 |
| 5107 | 5106 | " | 2553 | 9521 | 952 | " | 595 |
| 5407 | 901 | " | 1802 | 9629 | 4814 | " | 9628 |
| 5479 | 5478 | " | 2739 | 9649 | 1206 | " | 603 |
| 5519 | 5518 | " | 2759 | 9941 | 9940 | " | 1988 |

N.B. There are 64 errors, 3 misprints, 2 omissions, viz. 3 and 7853

TABLE II. List of Errors in Burekhardt's and Jacobi's Tables.

| | Primes. | No. of figures in period of reciprocal. | | No. of figures in period of reciprocal. |
|---------------------------|-------------|---|-----------|---|
| Burekhardt and Jacobi ... | 911 | 450 | should be | 455 |
| Burekhardt | 1979 | 1976 | " | 1978 |
| Jacobi | 3462 | | | |
| | should be } | | | |
| | 3467 | | | |
| | 1213 | 1212 | " | 202 |
| | 1597 | 266 | " | 133 |
| | 1831 | 915 | " | 305 |
| Burekhardt and Jacobi... | 1951 | 390 | " | 195 |
| | 1993 | 1992 | " | 664 |
| | 2311 | 462 | " | 231 |
| | 2437 | 2436 | " | 1218 |
| | 367 | 3466 | " | 1733 |

N.B. There are 3 misprints and 8 errors,

In the left-hand columns of Table III. are primes; in the right-hand columns, immediately opposite, is the number of figures in the period of the reciprocal of each prime.

TABLE III.

| | | | | | | | | | |
|-----|-----|-----|-----|------|------|------|------|------|------|
| 3 | 1 | 311 | 155 | 691 | 230 | 1109 | 1108 | 1567 | 1566 |
| 7 | 6 | 313 | 312 | 701 | 700 | 1117 | 558 | 1571 | 1570 |
| 11 | 2 | 317 | 79 | 709 | 708 | 1123 | 561 | 1579 | 1578 |
| 13 | 6 | 331 | 110 | 719 | 359 | 1129 | 564 | 1583 | 1582 |
| 17 | 16 | 337 | 336 | 727 | 726 | 1151 | 575 | 1597 | 1597 |
| 19 | 18 | 347 | 173 | 733 | 61 | 1153 | 1152 | 1601 | 200 |
| 23 | 22 | 349 | 116 | 739 | 246 | 1163 | 581 | 1607 | 1606 |
| 29 | 28 | 353 | 32 | 743 | 742 | 1171 | 1170 | 1609 | 201 |
| 31 | 15 | 359 | 179 | 751 | 125 | 1181 | 1180 | 1613 | 403 |
| 37 | 3 | 367 | 366 | 757 | 27 | 1187 | 593 | 1619 | 1618 |
| 41 | 5 | 373 | 186 | 761 | 380 | 1193 | 1192 | 1621 | 1620 |
| 43 | 21 | 379 | 378 | 769 | 192 | 1201 | 200 | 1627 | 271 |
| 47 | 46 | 383 | 382 | 773 | 193 | 1213 | 202 | 1637 | 409 |
| 53 | 13 | 389 | 388 | 787 | 393 | 1217 | 1216 | 1657 | 552 |
| 59 | 58 | 397 | 99 | 797 | 199 | 1223 | 1222 | 1663 | 1662 |
| 61 | 60 | 401 | 200 | 809 | 202 | 1229 | 1228 | 1667 | 833 |
| 67 | 33 | 409 | 204 | 811 | 810 | 1231 | 41 | 1669 | 556 |
| 71 | 35 | 419 | 418 | 821 | 820 | 1237 | 206 | 1693 | 423 |
| 73 | 8 | 421 | 140 | 823 | 822 | 1249 | 208 | 1697 | 1696 |
| 79 | 13 | 431 | 215 | 827 | 413 | 1259 | 1258 | 1699 | 566 |
| 83 | 41 | 433 | 432 | 829 | 276 | 1277 | 638 | 1709 | 1708 |
| 89 | 44 | 439 | 219 | 839 | 419 | 1279 | 639 | 1721 | 430 |
| 97 | 96 | 443 | 221 | 853 | 213 | 1283 | 641 | 1723 | 287 |
| 101 | 4 | 449 | 32 | 857 | 856 | 1289 | 92 | 1733 | 866 |
| 103 | 34 | 457 | 152 | 859 | 26 | 1291 | 1290 | 1741 | 1740 |
| 107 | 53 | 461 | 460 | 863 | 862 | 1297 | 1296 | 1747 | 291 |
| 109 | 108 | 463 | 154 | 877 | 438 | 1301 | 1300 | 1753 | 584 |
| 113 | 112 | 467 | 233 | 881 | 440 | 1303 | 1302 | 1759 | 879 |
| 127 | 42 | 479 | 239 | 883 | 441 | 1307 | 653 | 1777 | 1776 |
| 131 | 130 | 487 | 486 | 887 | 886 | 1319 | 659 | 1783 | 1782 |
| 137 | 8 | 491 | 490 | 907 | 151 | 1321 | 55 | 1787 | 893 |
| 139 | 46 | 499 | 498 | 911 | 455 | 1327 | 1326 | 1789 | 1788 |
| 149 | 148 | 503 | 502 | 919 | 459 | 1361 | 680 | 1801 | 900 |
| 151 | 75 | 509 | 508 | 929 | 464 | 1367 | 1366 | 1811 | 1810 |
| 157 | 78 | 521 | 52 | 937 | 936 | 1373 | 686 | 1823 | 1822 |
| 163 | 81 | 523 | 261 | 941 | 940 | 1381 | 1380 | 1831 | 305 |
| 167 | 166 | 541 | 540 | 947 | 473 | 1399 | 699 | 1847 | 1846 |
| 173 | 43 | 547 | 91 | 953 | 952 | 1409 | 32 | 1861 | 1860 |
| 179 | 178 | 557 | 278 | 967 | 322 | 1423 | 158 | 1867 | 933 |
| 181 | 180 | 563 | 281 | 971 | 970 | 1427 | 713 | 1871 | 935 |
| 191 | 95 | 569 | 284 | 977 | 976 | 1429 | 1428 | 1873 | 1872 |
| 193 | 192 | 571 | 570 | 983 | 982 | 1433 | 1432 | 1877 | 938 |
| 197 | 98 | 577 | 576 | 991 | 495 | 1439 | 719 | 1879 | 313 |
| 199 | 99 | 587 | 293 | 997 | 166 | 1447 | 1446 | 1889 | 118 |
| 211 | 30 | 593 | 592 | 1009 | 252 | 1451 | 290 | 1901 | 380 |
| 223 | 222 | 599 | 299 | 1013 | 253 | 1453 | 726 | 1907 | 953 |
| 227 | 113 | 601 | 300 | 1019 | 1018 | 1459 | 162 | 1913 | 1912 |
| 229 | 228 | 607 | 202 | 1021 | 1020 | 1471 | 735 | 1931 | 386 |
| 233 | 232 | 613 | 51 | 1031 | 103 | 1481 | 740 | 1933 | 21 |
| 239 | 7 | 617 | 88 | 1033 | 1032 | 1483 | 247 | 1949 | 1948 |
| 241 | 30 | 619 | 618 | 1039 | 519 | 1487 | 1486 | 1951 | 195 |
| 251 | 50 | 631 | 315 | 1049 | 524 | 1489 | 248 | 1973 | 986 |
| 257 | 256 | 641 | 32 | 1051 | 1050 | 1493 | 373 | 1979 | 1978 |
| 263 | 262 | 643 | 107 | 1061 | 212 | 1499 | 214 | 1987 | 331 |
| 269 | 268 | 647 | 646 | 1063 | 1062 | 1511 | 755 | 1993 | 664 |
| 271 | 5 | 653 | 326 | 1069 | 1068 | 1523 | 761 | 1997 | 998 |
| 277 | 69 | 659 | 658 | 1087 | 1086 | 1531 | 1530 | 1999 | 999 |
| 281 | 28 | 661 | 220 | 1091 | 1090 | 1543 | 1542 | 2003 | 1001 |
| 283 | 141 | 673 | 224 | 1093 | 273 | 1549 | 1548 | 2011 | 670 |
| 293 | 146 | 677 | 338 | 1097 | 1096 | 1553 | 1552 | 2017 | 2016 |
| 307 | 153 | 683 | 341 | 1103 | 1102 | 1559 | 779 | 2027 | 1013 |

TABLE III. (continued).

| | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|
| 2029 | 2028 | 2539 | 2538 | 3023 | 3022 | 3547 | 1773 | 4057 | 4056 |
| 2039 | 1019 | 2543 | 2542 | 3037 | 253 | 3557 | 254 | 4073 | 4072 |
| 2053 | 342 | 2549 | 2548 | 3041 | 380 | 3559 | 1779 | 4079 | 2039 |
| 2063 | 2062 | 2551 | 425 | 3049 | 508 | 3571 | 3570 | 4091 | 4090 |
| 2069 | 2068 | 2557 | 639 | 3061 | 204 | 3581 | 3580 | 4093 | 22 |
| 2081 | 1040 | 2579 | 2578 | 3067 | 1533 | 3583 | 1194 | 4099 | 4098 |
| 2083 | 1041 | 2591 | 259 | 3079 | 1539 | 3593 | 3592 | 4111 | 2055 |
| 2087 | 208 | 2593 | 2592 | 3083 | 1541 | 3607 | 3606 | 4127 | 4126 |
| 2089 | 1044 | 2609 | 1304 | 3089 | 1544 | 3613 | 602 | 4129 | 2064 |
| 2099 | 2098 | 2617 | 2616 | 3109 | 148 | 3617 | 3616 | 4133 | 1033 |
| 2111 | 1055 | 2621 | 2620 | 3119 | 1559 | 3623 | 3622 | 4139 | 4138 |
| 2113 | 2112 | 2633 | 2632 | 3121 | 156 | 3631 | 1815 | 4153 | 4152 |
| 2129 | 532 | 2647 | 882 | 3137 | 3136 | 3637 | 909 | 4157 | 2078 |
| 2131 | 710 | 2657 | 2656 | 3163 | 1581 | 3643 | 1821 | 4159 | 693 |
| 2137 | 2136 | 2659 | 886 | 3167 | 3166 | 3659 | 3658 | 4177 | 4176 |
| 2141 | 2140 | 2663 | 2662 | 3169 | 72 | 3671 | 367 | 4201 | 75 |
| 2143 | 2142 | 2670 | 1335 | 3181 | 636 | 3673 | 3672 | 4211 | 4210 |
| 2153 | 2152 | 2677 | 223 | 3187 | 177 | 3677 | 1838 | 4217 | 4216 |
| 2161 | 30 | 2683 | 447 | 3191 | 29 | 3691 | 1230 | 4219 | 4218 |
| 2179 | 2178 | 2687 | 2686 | 3203 | 1601 | 3697 | 1232 | 4229 | 4228 |
| 2203 | 1101 | 2689 | 42 | 3209 | 1604 | 3701 | 3700 | 4231 | 2115 |
| 2207 | 2206 | 2693 | 1346 | 3217 | 1072 | 3709 | 3708 | 4241 | 1060 |
| 2213 | 553 | 2699 | 2698 | 3221 | 3220 | 3719 | 1859 | 4243 | 2121 |
| 2221 | 2220 | 2707 | 1353 | 3229 | 1076 | 3727 | 3726 | 4253 | 1063 |
| 2237 | 1118 | 2711 | 1355 | 3251 | 3250 | 3733 | 933 | 4259 | 4258 |
| 2239 | 1119 | 2713 | 2712 | 3253 | 542 | 3739 | 1246 | 4261 | 4260 |
| 2243 | 1121 | 2719 | 1359 | 3257 | 3256 | 3761 | 1880 | 4271 | 2135 |
| 2251 | 2250 | 2729 | 682 | 3259 | 3258 | 3767 | 3766 | 4273 | 1424 |
| 2267 | 1133 | 2731 | 2730 | 3271 | 1635 | 3769 | 1884 | 4283 | 2141 |
| 2269 | 2268 | 2741 | 2740 | 3299 | 3298 | 3779 | 3778 | 4289 | 2144 |
| 2273 | 2272 | 2749 | 916 | 3301 | 3300 | 3793 | 1264 | 4297 | 1432 |
| 2281 | 228 | 2753 | 2752 | 3307 | 1653 | 3797 | 949 | 4327 | 4326 |
| 2287 | 762 | 2767 | 2766 | 3313 | 3312 | 3803 | 1901 | 4337 | 4336 |
| 2293 | 1146 | 2777 | 2776 | 3319 | 553 | 3821 | 3820 | 4339 | 4338 |
| 2297 | 2296 | 2789 | 2788 | 3323 | 1661 | 3823 | 1274 | 4349 | 4348 |
| 2309 | 2308 | 2791 | 31 | 3329 | 832 | 3833 | 3832 | 4357 | 242 |
| 2311 | 231 | 2797 | 699 | 3331 | 3330 | 3847 | 3846 | 4363 | 2181 |
| 2333 | 583 | 2801 | 1400 | 3343 | 3342 | 3851 | 770 | 4373 | 1093 |
| 2339 | 2338 | 2803 | 1401 | 3347 | 1673 | 3853 | 963 | 4391 | 2195 |
| 2341 | 2340 | 2819 | 2818 | 3359 | 1679 | 3863 | 3862 | 4397 | 314 |
| 2347 | 1173 | 2833 | 2832 | 3361 | 1680 | 3877 | 969 | 4409 | 551 |
| 2351 | 1175 | 2837 | 709 | 3371 | 3370 | 3881 | 1940 | 4421 | 4420 |
| 2357 | 1178 | 2843 | 1421 | 3373 | 843 | 3889 | 1944 | 4423 | 4422 |
| 2371 | 2370 | 2851 | 2850 | 3389 | 3388 | 3907 | 1953 | 4441 | 2220 |
| 2377 | 264 | 2857 | 408 | 3391 | 1695 | 3911 | 1955 | 4447 | 4446 |
| 2381 | 476 | 2861 | 2860 | 3407 | 3406 | 3917 | 1958 | 4451 | 4450 |
| 2383 | 2382 | 2879 | 1439 | 3413 | 1706 | 3919 | 653 | 4457 | 4456 |
| 2389 | 2388 | 2887 | 2886 | 3433 | 3432 | 3923 | 1961 | 4463 | 4462 |
| 2393 | 184 | 2897 | 2896 | 3449 | 431 | 3929 | 491 | 4481 | 2240 |
| 2399 | 1199 | 2903 | 2902 | 3457 | 384 | 3931 | 1310 | 4483 | 249 |
| 2411 | 2410 | 2909 | 2908 | 3461 | 3460 | 3943 | 3942 | 4493 | 1123 |
| 2417 | 2416 | 2917 | 1458 | 3463 | 3462 | 3947 | 1973 | 4507 | 751 |
| 2423 | 2422 | 2927 | 2926 | 3467 | 1733 | 3967 | 3966 | 4513 | 1504 |
| 2437 | 1218 | 2939 | 2938 | 3469 | 3468 | 3989 | 3988 | 4517 | 2058 |
| 2441 | 305 | 2953 | 984 | 3491 | 698 | 4001 | 500 | 4519 | 753 |
| 2447 | 2446 | 2957 | 1478 | 3499 | 318 | 4003 | 87 | 4523 | 2261 |
| 2459 | 2458 | 2963 | 1481 | 3511 | 1755 | 4007 | 4006 | 4547 | 2273 |
| 2467 | 137 | 2969 | 371 | 3517 | 879 | 4013 | 34 | 4549 | 1516 |
| 2473 | 2472 | 2971 | 2970 | 3527 | 3526 | 4019 | 4018 | 4561 | 2280 |
| 2477 | 619 | 2999 | 1499 | 3529 | 1764 | 4021 | 268 | 4567 | 4566 |
| 2503 | 278 | 3001 | 1500 | 3533 | 1766 | 4027 | 2013 | 4583 | 4582 |
| 2521 | 630 | 3011 | 3010 | 3539 | 3538 | 4049 | 2024 | 4591 | 2295 |
| 2531 | 46 | 3019 | 3018 | 3541 | 20 | 4051 | 4050 | 4597 | 2298 |

TABLE III. (continued).

| | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|
| 4603 | 2301 | 5147 | 2573 | 5689 | 316 | 6247 | 6246 | 6803 | 3401 |
| 4621 | 924 | 5153 | 5152 | 5693 | 1423 | 6257 | 6256 | 6823 | 6822 |
| 4637 | 61 | 5167 | 5166 | 5701 | 5700 | 6263 | 6262 | 6827 | 3413 |
| 4639 | 2319 | 5171 | 110 | 5711 | 571 | 6269 | 6268 | 6829 | 6828 |
| 4643 | 2321 | 5179 | 5178 | 5717 | 1429 | 6271 | 1045 | 6833 | 6832 |
| 4649 | 7 | 5189 | 5188 | 5737 | 5736 | 6277 | 1569 | 6841 | 855 |
| 4651 | 4650 | 5197 | 433 | 5741 | 5740 | 6287 | 6286 | 6857 | 6856 |
| 4657 | 1552 | 5209 | 372 | 5743 | 5742 | 6299 | 94 | 6863 | 6862 |
| 4663 | 222 | 5227 | 2613 | 5749 | 5748 | 6301 | 6300 | 6869 | 6868 |
| 4673 | 4672 | 5231 | 2615 | 5779 | 2889 | 6317 | 3158 | 6871 | 3435 |
| 4679 | 2339 | 5233 | 5232 | 5783 | 5782 | 6323 | 3161 | 6883 | 3441 |
| 4691 | 4690 | 5237 | 77 | 5791 | 965 | 6329 | 3164 | 6899 | 6898 |
| 4703 | 4702 | 5261 | 1052 | 5801 | 1450 | 6337 | 6336 | 6907 | 1151 |
| 4721 | 2360 | 5273 | 5272 | 5807 | 5806 | 6343 | 6342 | 6911 | 3455 |
| 4723 | 2361 | 5279 | 2639 | 5813 | 2906 | 6353 | 6352 | 6947 | 3473 |
| 4729 | 1182 | 5281 | 2640 | 5821 | 5820 | 6359 | 3179 | 6949 | 6948 |
| 4733 | 1183 | 5297 | 5296 | 5827 | 2913 | 6361 | 1590 | 6959 | 3479 |
| 4751 | 2375 | 5303 | 5302 | 5839 | 2919 | 6367 | 6366 | 6961 | 3480 |
| 4759 | 2379 | 5309 | 5308 | 5843 | 2921 | 6373 | 1062 | 6967 | 6966 |
| 4783 | 4782 | 5323 | 2661 | 5849 | 1462 | 6379 | 2126 | 6971 | 6970 |
| 4787 | 2393 | 5333 | 1333 | 5851 | 1950 | 6389 | 6388 | 6977 | 6976 |
| 4789 | 228 | 5347 | 2673 | 5857 | 5856 | 6397 | 78 | 6983 | 6982 |
| 4793 | 4792 | 5351 | 2675 | 5861 | 5860 | 6421 | 2140 | 6991 | 3495 |
| 4799 | 2399 | 5381 | 5380 | 5867 | 2933 | 6427 | 1071 | 6997 | 1749 |
| 4801 | 800 | 5387 | 2693 | 5869 | 5868 | 6449 | 1612 | 7001 | 1750 |
| 4813 | 802 | 5393 | 5392 | 5879 | 2939 | 6451 | 2150 | 7013 | 3506 |
| 4817 | 4816 | 5399 | 2699 | 5881 | 2940 | 6469 | 924 | 7019 | 7018 |
| 4831 | 805 | 5407 | 1802 | 5897 | 5896 | 6473 | 6472 | 7027 | 1171 |
| 4861 | 972 | 5413 | 2706 | 5903 | 5902 | 6481 | 270 | 7039 | 391 |
| 4871 | 2435 | 5417 | 5416 | 5923 | 2961 | 6491 | 1298 | 7043 | 503 |
| 4877 | 1219 | 5419 | 5418 | 5927 | 5926 | 6521 | 815 | 7057 | 7056 |
| 4889 | 2444 | 5431 | 2715 | 5939 | 5938 | 6529 | 1088 | 7069 | 7068 |
| 4903 | 1634 | 5437 | 1359 | 5953 | 1984 | 6547 | 1091 | 7079 | 3539 |
| 4909 | 1636 | 5441 | 2720 | 5981 | 5980 | 6551 | 3275 | 7103 | 7102 |
| 4919 | 2459 | 5443 | 907 | 5987 | 2993 | 6553 | 6552 | 7109 | 7108 |
| 4931 | 4930 | 5449 | 2724 | 6007 | 858 | 6563 | 3281 | 7121 | 3560 |
| 4933 | 2466 | 5471 | 547 | 6011 | 6010 | 6569 | 1642 | 7127 | 1018 |
| 4937 | 4936 | 5477 | 1369 | 6029 | 6028 | 6571 | 6570 | 7129 | 594 |
| 4943 | 4942 | 5479 | 2739 | 6037 | 3018 | 6577 | 2192 | 7151 | 275 |
| 4951 | 2475 | 5483 | 2741 | 6043 | 3021 | 6581 | 1316 | 7159 | 3579 |
| 4957 | 413 | 5501 | 5500 | 6047 | 6046 | 6599 | 3299 | 7177 | 7176 |
| 4967 | 4966 | 5503 | 5502 | 6053 | 3026 | 6607 | 2202 | 7187 | 3593 |
| 4969 | 828 | 5507 | 2753 | 6067 | 3033 | 6619 | 6618 | 7193 | 7192 |
| 4973 | 226 | 5519 | 2759 | 6073 | 6072 | 6637 | 474 | 7207 | 7206 |
| 4987 | 2493 | 5521 | 345 | 6079 | 1013 | 6653 | 3326 | 7211 | 1030 |
| 4993 | 1664 | 5527 | 5526 | 6089 | 761 | 6659 | 6658 | 7213 | 1803 |
| 4999 | 357 | 5531 | 5530 | 6091 | 2030 | 6661 | 6660 | 7219 | 7218 |
| 5003 | 2501 | 5557 | 926 | 6101 | 1220 | 6673 | 6672 | 7229 | 7228 |
| 5009 | 626 | 5563 | 2781 | 6113 | 6112 | 6679 | 3339 | 7237 | 402 |
| 5011 | 1670 | 5569 | 1392 | 6121 | 3060 | 6689 | 1672 | 7243 | 3621 |
| 5021 | 5020 | 5573 | 2786 | 6131 | 6130 | 6691 | 6690 | 7247 | 7246 |
| 5023 | 1674 | 5581 | 5580 | 6133 | 1533 | 6701 | 7253 | 7251 | 74 |
| 5039 | 2519 | 5591 | 2795 | 6143 | 6142 | 6703 | 6702 | 7283 | 3641 |
| 5051 | 50 | 5623 | 5622 | 6151 | 1025 | 6709 | 6708 | 7297 | 2432 |
| 5059 | 5058 | 5639 | 2819 | 6163 | 79 | 6719 | 3359 | 7307 | 3053 |
| 5077 | 2538 | 5641 | 470 | 6173 | 3086 | 6733 | 3366 | 7309 | 7308 |
| 5081 | 1270 | 5647 | 1882 | 6197 | 3098 | 6737 | 6736 | 7321 | 3660 |
| 5087 | 5086 | 5651 | 5650 | 6199 | 3099 | 6761 | 1690 | 7331 | 1466 |
| 5099 | 5098 | 5653 | 2826 | 6203 | 443 | 6763 | 161 | 7333 | 611 |
| 5101 | 1700 | 5657 | 5656 | 6211 | 6210 | 6779 | 6778 | 7349 | 7348 |
| 5107 | 2553 | 5659 | 5658 | 6217 | 6216 | 6781 | 1356 | 7351 | 1225 |
| 5113 | 1704 | 5669 | 5668 | 6221 | 6220 | 6791 | 679 | 7369 | 1842 |
| 5119 | 853 | 5683 | 2841 | 6229 | 2076 | 6793 | 6792 | 7393 | 7392 |

TABLE III. (*continued*).

| | | | | | | | | | |
|------|------|------|------|------|------|------|------|-------|-------|
| 7411 | 7410 | 7937 | 7936 | 8543 | 8542 | 9103 | 9102 | 9649 | 603 |
| 7417 | 2472 | 7949 | 7948 | 8563 | 4281 | 9109 | 9108 | 9661 | 1380 |
| 7433 | 7432 | 7951 | 3975 | 8573 | 4286 | 9127 | 3042 | 9677 | 2419 |
| 7451 | 7450 | 7963 | 3981 | 8581 | 2860 | 9133 | 1522 | 9679 | 1613 |
| 7457 | 7456 | 7993 | 2664 | 8597 | 2149 | 9137 | 9136 | 9689 | 346 |
| 7459 | 7458 | 8009 | 2002 | 8599 | 1433 | 9151 | 1525 | 9697 | 9696 |
| 7477 | 3738 | 8011 | 2670 | 8609 | 1076 | 9157 | 4573 | 9719 | 4859 |
| 7481 | 748 | 8017 | 8016 | 8623 | 8622 | 9161 | 229 | 9721 | 4860 |
| 7487 | 7486 | 8039 | 4019 | 8627 | 4313 | 9173 | 4586 | 9733 | 2433 |
| 7489 | 1872 | 8053 | 4026 | 8629 | 2876 | 9181 | 3060 | 9739 | 9738 |
| 7499 | 7498 | 8059 | 8058 | 8641 | 4320 | 9187 | 4593 | 9743 | 9742 |
| 7507 | 3753 | 8069 | 8068 | 8647 | 8646 | 9199 | 4599 | 9749 | 9748 |
| 7517 | 3758 | 8081 | 2020 | 8663 | 8662 | 9203 | 4601 | 9767 | 9766 |
| 7523 | 3761 | 8087 | 8086 | 8669 | 8668 | 9209 | 2302 | 9769 | 4884 |
| 7529 | 1882 | 8089 | 1348 | 8677 | 723 | 9221 | 9220 | 9781 | 9780 |
| 7537 | 2512 | 8093 | 4046 | 8681 | 868 | 9227 | 4613 | 9787 | 4893 |
| 7541 | 7540 | 8101 | 1620 | 8689 | 2172 | 9239 | 4619 | 9791 | 4895 |
| 7547 | 3773 | 8111 | 811 | 8693 | 4346 | 9241 | 4620 | 9803 | 4901 |
| 7549 | 2516 | 8117 | 2029 | 8699 | 8698 | 9257 | 9256 | 9811 | 9810 |
| 7559 | 3779 | 8123 | 4061 | 8707 | 4353 | 9277 | 4638 | 9817 | 9816 |
| 7561 | 1890 | 8147 | 4073 | 8713 | 8712 | 9281 | 928 | 9829 | 9828 |
| 7573 | 631 | 8161 | 1020 | 8719 | 4359 | 9283 | 1547 | 9833 | 9832 |
| 7577 | 7576 | 8167 | 2722 | 8731 | 8730 | 9293 | 2323 | 9839 | 4919 |
| 7583 | 7582 | 8171 | 8170 | 8737 | 2912 | 9311 | 4655 | 9851 | 9850 |
| 7589 | 1084 | 8179 | 8178 | 8741 | 8740 | 9319 | 4659 | 9857 | 9856 |
| 7591 | 3795 | 8191 | 1365 | 8747 | 4373 | 9323 | 4661 | 9859 | 3286 |
| 7603 | 1267 | 8209 | 4104 | 8753 | 8752 | 9337 | 3112 | 9871 | 4935 |
| 7607 | 7606 | 8219 | 8218 | 8761 | 876 | 9341 | 9340 | 9883 | 4941 |
| 7621 | 508 | 8221 | 2740 | 8779 | 22 | 9343 | 9342 | 9887 | 9886 |
| 7639 | 3819 | 8231 | 4115 | 8783 | 8782 | 9349 | 3116 | 9901 | 12 |
| 7643 | 3821 | 8233 | 8232 | 8803 | 1467 | 9371 | 9370 | 9907 | 4953 |
| 7649 | 1912 | 8237 | 4118 | 8807 | 8806 | 9377 | 9376 | 9923 | 4961 |
| 7669 | 284 | 8243 | 4121 | 8819 | 8818 | 9391 | 4695 | 9929 | 1241 |
| 7673 | 7672 | 8263 | 8262 | 8821 | 8820 | 9397 | 81 | 9931 | 9930 |
| 7681 | 1920 | 8269 | 8268 | 8831 | 4415 | 9403 | 1567 | 9941 | 1988 |
| 7687 | 7686 | 8273 | 8272 | 8837 | 4418 | 9413 | 4706 | 9949 | 9948 |
| 7691 | 7690 | 8287 | 8286 | 8839 | 4419 | 9419 | 554 | 9967 | 9966 |
| 7699 | 7698 | 8291 | 8290 | 8849 | 553 | 9421 | 9420 | 9973 | 554 |
| 7703 | 7702 | 8293 | 2073 | 8861 | 8860 | 9431 | 4715 | 10007 | 10006 |
| 7717 | 1929 | 8297 | 8296 | 8863 | 8862 | 9433 | 1048 | 10009 | 5004 |
| 7723 | 1287 | 8311 | 4155 | 8867 | 4433 | 9437 | 4718 | 10037 | 386 |
| 7727 | 7726 | 8317 | 462 | 8887 | 8886 | 9439 | 1573 | 10039 | 5019 |
| 7741 | 860 | 8329 | 1041 | 8893 | 2223 | 9461 | 9460 | 10061 | 10060 |
| 7753 | 7752 | 8353 | 8352 | 8923 | 1487 | 9463 | 3154 | 10067 | 5033 |
| 7757 | 1939 | 8363 | 4181 | 8929 | 144 | 9467 | 4733 | 10069 | 10068 |
| 7759 | 3879 | 8369 | 4184 | 8933 | 2233 | 9473 | 9472 | 10079 | 5039 |
| 7789 | 2596 | 8377 | 8376 | 8941 | 2980 | 9479 | 4739 | 10091 | 10090 |
| 7793 | 7792 | 8387 | 599 | 8951 | 4475 | 9491 | 9490 | 10093 | 2523 |
| 7817 | 7816 | 8389 | 8388 | 8963 | 4481 | 9497 | 9496 | 10099 | 3366 |
| 7823 | 7822 | 8419 | 2806 | 8969 | 4484 | 9511 | 1585 | 10103 | 10102 |
| 7829 | 7828 | 8423 | 8422 | 8971 | 8970 | 9521 | 595 | 10111 | 5055 |
| 7841 | 56 | 8429 | 8428 | 8999 | 4499 | 9533 | 2383 | 10133 | 2533 |
| 7853 | 3926 | 8431 | 4215 | 9001 | 1125 | 9539 | 9538 | 10139 | 10138 |
| 7867 | 3933 | 8443 | 4221 | 9007 | 3002 | 9547 | 4773 | 10141 | 10140 |
| 7873 | 7872 | 8447 | 8446 | 9011 | 9010 | 9551 | 955 | 10151 | 5075 |
| 7877 | 3938 | 8461 | 2820 | 9013 | 2253 | 9587 | 4793 | 10159 | 5079 |
| 7879 | 3939 | 8467 | 4233 | 9029 | 9028 | 9601 | 4800 | 10163 | 5081 |
| 7883 | 3941 | 8501 | 8500 | 9041 | 1130 | 9613 | 267 | 10169 | 5084 |
| 7901 | 7900 | 8513 | 8512 | 9043 | 4521 | 9619 | 3206 | 10177 | 10176 |
| 7907 | 3953 | 8521 | 710 | 9049 | 4524 | 9623 | 9622 | 10181 | 10180 |
| 7919 | 3959 | 8527 | 2842 | 9059 | 9058 | 9629 | 9628 | 10193 | 10192 |
| 7927 | 7926 | 8537 | 8536 | 9067 | 4533 | 9631 | 4815 | 10211 | 10210 |
| 7933 | 3966 | 8539 | 2846 | 9091 | 10 | 9643 | 4821 | 10223 | 10222 |

TABLE III. (continued).

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10243 | 569 | 10847 | 10846 | 11447 | 11446 | 12049 | 6024 | 12613 | 6306 |
| 10247 | 10246 | 10853 | 5426 | 11467 | 5733 | 12071 | 355 | 12619 | 4206 |
| 10253 | 2563 | 10859 | 10858 | 11471 | 5735 | 12073 | 12072 | 12637 | 3159 |
| 10259 | 10258 | 10861 | 10860 | 11483 | 5741 | 12097 | 4032 | 12641 | 3160 |
| 10267 | 5133 | 10867 | 1811 | 11489 | 2872 | 12101 | 12100 | 12647 | 12646 |
| 10271 | 79 | 10883 | 5441 | 11491 | 766 | 12107 | 6053 | 12653 | 6326 |
| 10273 | 10272 | 10889 | 2722 | 11497 | 11496 | 12109 | 4036 | 12659 | 12658 |
| 10289 | 5144 | 10891 | 1210 | 11503 | 11502 | 12113 | 12112 | 12671 | 181 |
| 10301 | 10300 | 10903 | 10902 | 11519 | 5759 | 12119 | 6059 | 12689 | 793 |
| 10303 | 3434 | 10909 | 1212 | 11527 | 3842 | 12143 | 12142 | 12697 | 12696 |
| 10313 | 10312 | 10937 | 10936 | 11549 | 11548 | 12149 | 12148 | 12703 | 4234 |
| 10321 | 2580 | 10939 | 10938 | 11551 | 1925 | 12157 | 2026 | 12713 | 12712 |
| 10331 | 2066 | 10949 | 10948 | 11579 | 11578 | 12161 | 6080 | 12721 | 2120 |
| 10333 | 5166 | 10957 | 2739 | 11587 | 1931 | 12163 | 6081 | 12739 | 4246 |
| 10337 | 10336 | 10973 | 2743 | 11593 | 11592 | 12197 | 3049 | 12743 | 12742 |
| 10343 | 10342 | 10979 | 10978 | 11597 | 5798 | 12203 | 6101 | 12757 | 2126 |
| 10357 | 5178 | 10987 | 5493 | 11617 | 11616 | 12211 | 4070 | 12763 | 709 |
| 10369 | 2592 | 10993 | 10992 | 11621 | 12227 | 6113 | 12781 | 12780 | |
| 10391 | 5195 | 11003 | 5501 | 11633 | 11632 | 12239 | 6119 | 12791 | 6395 |
| 10399 | 1733 | 11027 | 5513 | 11657 | 11656 | 12241 | 6120 | 12799 | 2133 |
| 10427 | 5213 | 11047 | 11046 | 11677 | 5838 | 12251 | 12250 | 12809 | 6404 |
| 10429 | 948 | 11057 | 11056 | 11681 | 5840 | 12253 | 3063 | 12821 | 12820 |
| 10433 | 10432 | 11059 | 11058 | 11689 | 487 | 12263 | 12262 | 12823 | 12822 |
| 10453 | 5226 | 11069 | 11068 | 11699 | 11698 | 12269 | 12268 | 12829 | 4276 |
| 10457 | 10456 | 11071 | 615 | 11701 | 11700 | 12277 | 3069 | 12841 | 6420 |
| 10459 | 10458 | 11083 | 1847 | 11717 | 2929 | 12281 | 6140 | 12853 | 459 |
| 10463 | 10462 | 11087 | 482 | 11719 | 5859 | 12289 | 384 | 12889 | 3222 |
| 10477 | 1746 | 11093 | 2773 | 11731 | 11730 | 12301 | 2460 | 12893 | 3223 |
| 10487 | 10486 | 11113 | 3704 | 11743 | 11742 | 12323 | 6161 | 12899 | 12898 |
| 10499 | 10498 | 11117 | 2779 | 11777 | 11776 | 12329 | 3082 | 12907 | 6453 |
| 10501 | 3500 | 11119 | 5559 | 11779 | 3926 | 12343 | 4114 | 12911 | 6455 |
| 10513 | 10512 | 11131 | 11130 | 11783 | 11782 | 12347 | 6173 | 12917 | 6458 |
| 10529 | 5264 | 11149 | 11148 | 11789 | 11788 | 12373 | 6186 | 12919 | 2153 |
| 10531 | 10530 | 11159 | 5579 | 11801 | 2950 | 12377 | 12376 | 12923 | 6461 |
| 10559 | 5279 | 11161 | 310 | 11807 | 11806 | 12379 | 12378 | 12941 | 12940 |
| 10567 | 10566 | 11171 | 11170 | 11813 | 5906 | 12391 | 6195 | 12953 | 12952 |
| 10589 | 10588 | 11173 | 5586 | 11821 | 11820 | 12401 | 12400 | 12959 | 6479 |
| 10597 | 5298 | 11177 | 11176 | 11827 | 5913 | 12409 | 6204 | 12967 | 4322 |
| 10601 | 10600 | 11197 | 2799 | 11831 | 169 | 12413 | 6206 | 12973 | 2162 |
| 10607 | 10606 | 11213 | 2803 | 11833 | 11832 | 12421 | 12420 | 12979 | 12978 |
| 10613 | 758 | 11239 | 5619 | 11839 | 5919 | 12433 | 4144 | 12983 | 12982 |
| 10627 | 5313 | 11243 | 5621 | 11863 | 11862 | 12437 | 6218 | 13001 | 1625 |
| 10631 | 5315 | 11251 | 2250 | 11867 | 5933 | 12451 | 12450 | 13003 | 6501 |
| 10639 | 5319 | 11257 | 11256 | 11887 | 11886 | 12457 | 12456 | 13007 | 13006 |
| 10651 | 10650 | 11261 | 2252 | 11897 | 11896 | 12473 | 12472 | 13009 | 2168 |
| 10657 | 10656 | 11273 | 11272 | 11903 | 11902 | 12479 | 6239 | 13033 | 4344 |
| 10663 | 10662 | 11279 | 5639 | 11909 | 11908 | 12487 | 12486 | 13037 | 6518 |
| 10667 | 5333 | 11287 | 11286 | 11923 | 5961 | 12491 | 12490 | 13043 | 6521 |
| 10687 | 10686 | 11299 | 11298 | 11927 | 11926 | 12497 | 12496 | 13049 | 3262 |
| 10691 | 10690 | 11311 | 377 | 11933 | 5966 | 12503 | 12502 | 13063 | 13062 |
| 10709 | 10708 | 11317 | 943 | 11939 | 11938 | 12511 | 2085 | 13093 | 2182 |
| 10711 | 595 | 11321 | 1132 | 11941 | 11940 | 12517 | 149 | 13099 | 13098 |
| 10723 | 5361 | 11329 | 1888 | 11953 | 11952 | 12527 | 12526 | 13103 | 13102 |
| 10729 | 596 | 11351 | 5675 | 11959 | 5979 | 12539 | 12538 | 13109 | 13108 |
| 10733 | 2683 | 11353 | 11352 | 11969 | 352 | 12541 | 4180 | 13121 | 6560 |
| 10739 | 10738 | 11369 | 812 | 11971 | 11970 | 12547 | 6273 | 13127 | 13126 |
| 10753 | 3584 | 11383 | 11382 | 11981 | 11980 | 12553 | 12552 | 13147 | 939 |
| 10771 | 2154 | 11393 | 11392 | 11987 | 5993 | 12569 | 6284 | 13151 | 1315 |
| 10781 | 10780 | 11399 | 5699 | 12007 | 4002 | 12577 | 12576 | 13159 | 2193 |
| 10789 | 10788 | 11411 | 2282 | 12011 | 12010 | 12583 | 12582 | 13163 | 6581 |
| 10799 | 5399 | 11423 | 11422 | 12037 | 3009 | 12589 | 12588 | 13171 | 4390 |
| 10831 | 5415 | 11437 | 2859 | 12041 | 6020 | 12601 | 6300 | 13177 | 13176 |
| 10837 | 63 | 11443 | 1907 | 12043 | 2007 | 12611 | 12610 | 13183 | 4394 |

TABLE III. (*continued*).

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 13187 | 6593 | 13807 | 13806 | 14461 | 4820 | 15061 | 5020 | 15629 | 15628 |
| 13217 | 13216 | 13829 | 13828 | 14479 | 7239 | 15073 | 15072 | 15641 | 391 |
| 13219 | 4406 | 13831 | 2305 | 14489 | 7244 | 15077 | 7538 | 15643 | 7821 |
| 13229 | 13228 | 13841 | 6920 | 14503 | 14502 | 15083 | 7541 | 15647 | 15646 |
| 13241 | 1655 | 13859 | 13858 | 14519 | 7259 | 15091 | 3018 | 15649 | 489 |
| 13249 | 288 | 13873 | 13872 | 14533 | 519 | 15101 | 604 | 15661 | 5220 |
| 13259 | 1894 | 13877 | 3469 | 14537 | 14536 | 15107 | 7553 | 15667 | 7833 |
| 13267 | 737 | 13879 | 6939 | 14543 | 14542 | 15121 | 7560 | 15671 | 1567 |
| 13291 | 13290 | 13883 | 6941 | 14549 | 14548 | 15131 | 3026 | 15679 | 2613 |
| 13297 | 13296 | 13901 | 13900 | 14551 | 485 | 15137 | 15136 | 15683 | 7841 |
| 13309 | 4436 | 13903 | 13902 | 14557 | 2426 | 15139 | 15138 | 15727 | 15726 |
| 13313 | 13312 | 13907 | 409 | 14561 | 7280 | 15149 | 15148 | 15731 | 15730 |
| 13327 | 4442 | 13913 | 13912 | 14563 | 809 | 15161 | 3790 | 15733 | 2622 |
| 13331 | 13330 | 13921 | 696 | 14591 | 7295 | 15173 | 7586 | 15737 | 15736 |
| 13337 | 13336 | 13931 | 13930 | 14593 | 14592 | 15187 | 7593 | 15739 | 15738 |
| 13339 | 13338 | 13933 | 6966 | 14621 | 14620 | 15193 | 15192 | 15749 | 15748 |
| 13367 | 13366 | 13963 | 6981 | 14627 | 7313 | 15199 | 7599 | 15761 | 394 |
| 13381 | 13380 | 13967 | 13966 | 14629 | 14628 | 15217 | 15216 | 15767 | 15766 |
| 13397 | 6698 | 13997 | 6998 | 14633 | 14632 | 15227 | 7613 | 15773 | 7886 |
| 13399 | 957 | 13999 | 2333 | 14639 | 7319 | 15233 | 15232 | 15787 | 7893 |
| 13411 | 13410 | 14009 | 3502 | 14653 | 3663 | 15241 | 3810 | 15791 | 7895 |
| 13417 | 4472 | 14011 | 14010 | 14657 | 14656 | 15259 | 15258 | 15797 | 3949 |
| 13421 | 13420 | 14029 | 14028 | 14669 | 14668 | 15203 | 15262 | 15803 | 7901 |
| 13441 | 6720 | 14033 | 14032 | 14683 | 2447 | 15269 | 1388 | 15809 | 3952 |
| 13451 | 13450 | 14051 | 2810 | 14699 | 14698 | 15271 | 7635 | 15817 | 5272 |
| 13457 | 13456 | 14057 | 14056 | 14713 | 14712 | 15277 | 3819 | 15823 | 15822 |
| 13463 | 13462 | 14071 | 7035 | 14717 | 7358 | 15287 | 15286 | 15859 | 15858 |
| 13469 | 13468 | 14081 | 1760 | 14723 | 7361 | 15289 | 1274 | 15877 | 567 |
| 13477 | 6738 | 14083 | 7041 | 14731 | 14730 | 15299 | 15298 | 15881 | 7940 |
| 13487 | 13486 | 14087 | 14086 | 14737 | 14736 | 15307 | 7653 | 15887 | 15886 |
| 13499 | 13498 | 14107 | 2351 | 14741 | 14740 | 15313 | 5104 | 15889 | 3972 |
| 13513 | 4504 | 14143 | 14142 | 14747 | 7373 | 15319 | 7659 | 15901 | 15900 |
| 13523 | 6761 | 14149 | 524 | 14753 | 14752 | 15329 | 7664 | 15907 | 7953 |
| 13537 | 13536 | 14153 | 14152 | 14759 | 7379 | 15331 | 5110 | 15913 | 15912 |
| 13553 | 1936 | 14159 | 7079 | 14767 | 14766 | 15349 | 15348 | 15919 | 7959 |
| 13567 | 4522 | 14173 | 3543 | 14771 | 2110 | 15359 | 7679 | 15923 | 7961 |
| 13577 | 13576 | 14177 | 14176 | 14779 | 14778 | 15361 | 256 | 15937 | 5312 |
| 13591 | 1359 | 14197 | 91 | 14783 | 14782 | 15373 | 1281 | 15959 | 7979 |
| 13597 | 3399 | 14207 | 14206 | 14797 | 3699 | 15377 | 15376 | 15971 | 15970 |
| 13613 | 6806 | 14221 | 2844 | 14813 | 7406 | 15383 | 15382 | 15973 | 121 |
| 13619 | 1238 | 14243 | 7121 | 14821 | 4940 | 15391 | 7695 | 15991 | 7995 |
| 13627 | 6813 | 14249 | 7124 | 14827 | 2471 | 15401 | 275 | 16001 | 2000 |
| 13633 | 4544 | 14251 | 14250 | 14831 | 1483 | 15413 | 7706 | 16007 | 16006 |
| 13649 | 853 | 14281 | 1190 | 14843 | 7421 | 15427 | 7713 | 16033 | 16032 |
| 13669 | 13668 | 14293 | 1191 | 14851 | 990 | 15439 | 7719 | 16057 | 1784 |
| 13679 | 6839 | 14303 | 14302 | 14867 | 7433 | 15443 | 7721 | 16061 | 3212 |
| 13681 | 3420 | 14321 | 3580 | 14869 | 4956 | 15451 | 5150 | 16063 | 5354 |
| 13687 | 4562 | 14323 | 1023 | 14879 | 7439 | 15461 | 15460 | 16067 | 8033 |
| 13691 | 13690 | 14327 | 14326 | 14887 | 14886 | 15467 | 7733 | 16069 | 5356 |
| 13693 | 326 | 14341 | 14340 | 14891 | 14890 | 15473 | 15472 | 16073 | 16072 |
| 13697 | 13696 | 14347 | 7173 | 14897 | 14896 | 15493 | 7746 | 16087 | 5362 |
| 13709 | 13708 | 14369 | 449 | 14923 | 7461 | 15497 | 1192 | 16091 | 16090 |
| 13711 | 6855 | 14387 | 7193 | 14929 | 1866 | 15511 | 7755 | 16097 | 16096 |
| 13721 | 6860 | 14389 | 4796 | 14939 | 14938 | 15527 | 15526 | 16103 | 16102 |
| 13723 | 6861 | 14401 | 3600 | 14947 | 7473 | 15541 | 740 | 16111 | 1611 |
| 13729 | 3432 | 14407 | 686 | 14951 | 1495 | 15551 | 7755 | 16127 | 16126 |
| 13751 | 6875 | 14411 | 2882 | 14957 | 7478 | 15559 | 2593 | 16139 | 16138 |
| 13757 | 362 | 14419 | 4806 | 14969 | 7484 | 15569 | 7784 | 16141 | 3228 |
| 13759 | 6879 | 14423 | 14422 | 14983 | 4994 | 15581 | 15580 | 16183 | 16182 |
| 13763 | 6881 | 14431 | 555 | 15013 | 3753 | 15583 | 15582 | 16187 | 8093 |
| 13781 | 13780 | 14437 | 218 | 15017 | 15016 | 15601 | 390 | 16189 | 852 |
| 13789 | 13788 | 14447 | 14446 | 15031 | 7515 | 15607 | 15606 | 16193 | 16192 |
| 13799 | 6899 | 14449 | 3612 | 15053 | 3763 | 15619 | 15618 | 16217 | 16216 |

TABLE III. (*continued*).

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 16223 | 16222 | 16889 | 8444 | 17483 | 8741 | 18097 | 6032 | 18719 | 9359 |
| 16229 | 16228 | 16901 | 3380 | 17489 | 2186 | 18119 | 9059 | 18731 | 3746 |
| 16231 | 8115 | 16903 | 5634 | 17491 | 17490 | 18121 | 2265 | 18743 | 18742 |
| 16249 | 1354 | 16921 | 423 | 17497 | 17496 | 18127 | 18126 | 18749 | 18748 |
| 16253 | 8126 | 16927 | 5642 | 17509 | 5836 | 18131 | 490 | 18757 | 3126 |
| 16267 | 8133 | 16931 | 16930 | 17519 | 8759 | 18133 | 9066 | 18773 | 9386 |
| 16273 | 16272 | 16937 | 16936 | 17539 | 5846 | 18143 | 18142 | 18787 | 9393 |
| 16301 | 3260 | 16943 | 16942 | 17551 | 2925 | 18149 | 1396 | 18793 | 18792 |
| 16319 | 8159 | 16963 | 8481 | 17569 | 2928 | 18169 | 9084 | 18797 | 127 |
| 16333 | 4083 | 16979 | 16978 | 17573 | 4393 | 18181 | 6060 | 18803 | 9401 |
| 16339 | 16338 | 16981 | 16980 | 17579 | 17578 | 18191 | 9095 | 18839 | 9419 |
| 16349 | 16348 | 16987 | 8493 | 17581 | 17580 | 18199 | 9099 | 18859 | 18858 |
| 16361 | 1636 | 16993 | 16992 | 17597 | 4399 | 18211 | 18210 | 18869 | 18868 |
| 16363 | 8181 | 17011 | 17010 | 17599 | 2933 | 18217 | 18216 | 18899 | 18898 |
| 16369 | 264 | 17021 | 3404 | 17609 | 8804 | 18223 | 6074 | 18911 | 9455 |
| 16381 | 5460 | 17027 | 8513 | 17623 | 1958 | 18229 | 18228 | 18913 | 18912 |
| 16411 | 16410 | 17029 | 17028 | 17627 | 8813 | 18233 | 424 | 18917 | 4729 |
| 16417 | 16416 | 17033 | 17032 | 17657 | 17656 | 18251 | 18250 | 18919 | 9459 |
| 16421 | 16420 | 17041 | 2840 | 17659 | 5886 | 18253 | 9126 | 18947 | 9473 |
| 16427 | 8213 | 17047 | 5682 | 17669 | 17668 | 18257 | 2608 | 18959 | 9479 |
| 16433 | 16432 | 17053 | 1421 | 17681 | 1105 | 18269 | 18268 | 18973 | 153 |
| 16447 | 16446 | 17077 | 2846 | 17683 | 8841 | 18287 | 18286 | 18979 | 18978 |
| 16451 | 16450 | 17093 | 4273 | 17707 | 8853 | 18289 | 9144 | 19001 | 2375 |
| 16453 | 2742 | 17099 | 17098 | 17713 | 17712 | 18301 | 18300 | 19009 | 4752 |
| 16477 | 4119 | 17107 | 8553 | 17729 | 1108 | 18307 | 3051 | 19013 | 4753 |
| 16481 | 2060 | 17117 | 4279 | 17737 | 17736 | 18311 | 9155 | 19031 | 9515 |
| 16487 | 16486 | 17123 | 8561 | 17747 | 8873 | 18313 | 18312 | 19037 | 9518 |
| 16493 | 4123 | 17137 | 5712 | 17749 | 17748 | 18329 | 4582 | 19051 | 6350 |
| 16519 | 2753 | 17159 | 8579 | 17761 | 8880 | 18341 | 18340 | 19069 | 6356 |
| 16529 | 8264 | 17167 | 17166 | 17783 | 17782 | 18353 | 18352 | 19073 | 19072 |
| 16547 | 8273 | 17183 | 17182 | 17789 | 17788 | 18367 | 6122 | 19079 | 9539 |
| 16553 | 16552 | 17189 | 17188 | 17791 | 8895 | 18371 | 3674 | 19081 | 795 |
| 16561 | 8280 | 17191 | 8595 | 17807 | 17806 | 18379 | 18378 | 19087 | 6362 |
| 16567 | 16566 | 17203 | 2867 | 17827 | 8913 | 18397 | 9198 | 19121 | 9560 |
| 16573 | 8286 | 17207 | 17206 | 17837 | 91 | 18401 | 4600 | 19139 | 19138 |
| 16603 | 8301 | 17209 | 4302 | 17839 | 8919 | 18413 | 4603 | 19141 | 19140 |
| 16607 | 16606 | 17231 | 8615 | 17851 | 714 | 18427 | 3071 | 19157 | 4789 |
| 16619 | 16618 | 17239 | 8619 | 17863 | 17862 | 18433 | 6144 | 19163 | 9581 |
| 16631 | 8315 | 17257 | 17256 | 17881 | 2235 | 18439 | 9219 | 19181 | 19180 |
| 16633 | 5544 | 17291 | 17290 | 17891 | 17890 | 18443 | 9221 | 19183 | 6394 |
| 16649 | 8324 | 17293 | 1441 | 17903 | 17902 | 18451 | 6150 | 19207 | 19206 |
| 16651 | 3330 | 17299 | 1922 | 17909 | 17908 | 18457 | 18456 | 19211 | 3842 |
| 16657 | 16656 | 17317 | 1443 | 17911 | 2985 | 18461 | 18460 | 19213 | 9606 |
| 16661 | 16660 | 17321 | 8660 | 17921 | 8960 | 18481 | 1320 | 19219 | 19218 |
| 16673 | 16672 | 17327 | 17326 | 17923 | 8961 | 18493 | 4623 | 19231 | 9615 |
| 16691 | 16690 | 17333 | 8666 | 17929 | 8964 | 18503 | 18502 | 19237 | 4809 |
| 16693 | 2782 | 17341 | 5780 | 17939 | 17938 | 18517 | 3086 | 19249 | 3208 |
| 16699 | 16698 | 17351 | 8675 | 17957 | 8978 | 18521 | 9260 | 19259 | 19258 |
| 16703 | 16702 | 17359 | 2893 | 17959 | 8979 | 18523 | 9261 | 19267 | 9633 |
| 16729 | 697 | 17377 | 17376 | 17971 | 17970 | 18539 | 18538 | 19273 | 19272 |
| 16741 | 16740 | 17383 | 17382 | 17977 | 856 | 18541 | 3708 | 19289 | 4822 |
| 16747 | 8373 | 17387 | 8693 | 17981 | 17980 | 18553 | 6184 | 19301 | 3860 |
| 16759 | 931 | 17389 | 17388 | 17987 | 8993 | 18583 | 18582 | 19309 | 19308 |
| 16763 | 29 | 17393 | 17392 | 17989 | 5996 | 18587 | 9293 | 19319 | 9659 |
| 16787 | 8393 | 17401 | 4350 | 18013 | 9006 | 18593 | 18592 | 19333 | 4833 |
| 16811 | 3362 | 17417 | 17416 | 18041 | 220 | 18617 | 1432 | 19373 | 4843 |
| 16823 | 16822 | 17419 | 17418 | 18043 | 291 | 18637 | 9318 | 19379 | 19378 |
| 16829 | 16828 | 17431 | 8715 | 18047 | 2578 | 18661 | 18660 | 19381 | 1292 |
| 16831 | 8415 | 17443 | 8721 | 18049 | 1128 | 18671 | 1867 | 19387 | 3231 |
| 16843 | 401 | 17449 | 4362 | 18059 | 18058 | 18679 | 3113 | 19391 | 1385 |
| 16871 | 1205 | 17467 | 8733 | 18061 | 18060 | 18691 | 18690 | 19403 | 9701 |
| 16879 | 2813 | 17471 | 8735 | 18077 | 4519 | 18701 | 3740 | 19417 | 19416 |
| 16883 | 8441 | 17477 | 8738 | 18089 | 9044 | 18713 | 18712 | 19421 | 19420 |

TABLE III. (*continued*).

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 19423 | 3237 | 19501 | 780 | 19681 | 9840 | 19777 | 6952 | 19913 | 19912 |
| 19427 | 9713 | 19507 | 9753 | 19687 | 19686 | 19793 | 19792 | 19919 | 9959 |
| 19429 | 19428 | 19531 | 6510 | 19697 | 19696 | 19801 | 9900 | 19927 | 19926 |
| 19433 | 19432 | 19541 | 19540 | 19699 | 19698 | 19813 | 4953 | 19937 | 2848 |
| 19441 | 1620 | 19543 | 19542 | 19709 | 19708 | 19819 | 19818 | 19949 | 19948 |
| 19447 | 19446 | 19559 | 9779 | 19717 | 9858 | 19841 | 1984 | 19961 | 9980 |
| 19457 | 19456 | 19571 | 19570 | 19727 | 19726 | 19843 | 3307 | 19963 | 3327 |
| 19463 | 19462 | 19577 | 19576 | 19739 | 19738 | 19853 | 9926 | 19973 | 9986 |
| 19469 | 19468 | 19583 | 19582 | 19751 | 9875 | 19861 | 6620 | 19979 | 19978 |
| 19471 | 9735 | 19597 | 4899 | 19753 | 19752 | 19867 | 6622 | 19991 | 9995 |
| 19477 | 3246 | 19603 | 3267 | 19759 | 9879 | 19889 | 9944 | 19993 | 19992 |
| 19483 | 9741 | 19609 | 9804 | 19763 | 9881 | 19891 | 6630 | 19997 | 9998 |
| 19489 | 406 | 19661 | 19660 | | | | | | |

February 26, 1874.

JOSEPH DALTON HOOKER, C.B., President, in the Chair.

The following Papers were read :—

- I. “The Winds of Northern India, in relation to the Temperature and Vapour-constituent of the Atmosphere.” By HENRY F. BLANFORD, F.G.S., Meteorological Reporter to the Government of Bengal. Communicated by Major-General STRACHEY, R.E. Received May 25, 1873.

(Abstract.)

The object of this paper is to describe the normal wind-currents of Northern India, and their annual variation, and to trace out their origin and causes, so far as these can be discovered in the local physical changes of the atmosphere. After referring to the data on which his conclusions are based, the author goes on to describe the winds of the principal geographical regions of North India in detail.

PART I. *Description of Winds.*

1. *The Punjab*.—As a rule currents from the westward predominate on an average throughout the year ; and this is also found to be the case in other parts of North India. In the most northern part of the Punjab, westerly winds prevail in the cold and hot dry months, easterly in the rainy months. In the central districts northerly winds preponderate over southerly, having in the cold months a westerly tendency, but drawing round to the north-east as the hot weather comes on, while as the rainy season sets in the winds tend to east and south-east, returning to west after the rain ceases in September. In the southern part of the Punjab (and this is also the case in Sindh) easterly winds never prevail,